FIR Filter Design using Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach

[1]Rajib Kar, [1]Durbadal Mandal, [1]Dibbendu Roy, [2]Sakti Prasad Ghoshal
[1] Department of ECE, National Institute of Technology, Durgapur, India
rajibkarece@gmail.com, durbadal.bittu@gmail.com , dibbaroy@gmail.com
[2] Department of EE, National Institute of Technology, Durgapur, India
spghoshalnitdgp@gmail.com

Abstract— **This paper presents an alternative approach for the design of linear phase digital low pass FIR filter using Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSO-CFIWA). FIR filter design is a multi-modal optimization problem. The conventional gradient based optimization techniques are not efficient for digital filter design. Given the filter specification to be realized, PSO algorithm generates a set of filter coefficients and tries to meet the ideal frequency characteristic. In this paper, for the given problem, the realization of the FIR filters of different order has been performed. The simulation results have been compared with the well accepted evolutionary algorithm such as genetic algorithm (GA). The results justify that the proposed filter design approach using PSO-CFIWA outperforms to that of GA, not only in the accuracy of the designed filter but also in the convergence speed and solution quality.**

Index Terms— **FIR Filter; PSO; GA; Optimization; Magnitude Response; Convergence; Low Pass Filter**

I. Introduction

A digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. This is in contrast to the other major type of electronic filter, the analog filter, which is an electronic circuit operating on continuous-time analog signals. There are two major classes of digital filters namely, finite impulse response (FIR) filters and infinite impulse response (IIR) filters depending on the length of the impulse response [7]. FIR filter is an attractive choice because of the ease in design and stability. By designing the filter taps to be symmetrical about the center tap position, a FIR filter can be guaranteed to have linear phase. Finite impulse response (FIR) digital filters are known to have many desirable features such as guaranteed stability, the possibility of exact linear phase characteristic at all frequencies and digital implementation as non-recursive structures. Linear phase FIR filters are also required when time domain specifications are given [1]. The most frequently used method for the design of exact linear phase weighted Chebyshev FIR digital filter is the one based on the Remez-exchange algorithm proposed by Parks and McClellan [2]. Further improvements to their results have been reported in [3]. The main limitation of this procedure is that the relative values of the amplitude error in the frequency bands are specified by means of the weighting function, and not by the deviations themselves. Therefore, in case of designing low-pass filters with a given stop band deviation, given filter length and cutoff frequencies, the program have to be iterated many times [4]. Different heuristic optimization algorithms such as genetic algorithm (GA), simulated annealing algorithms etc. have been widely used for the optimal design of digital filters. When considering global optimization methods for digital filter design, the GA seems to have attracted considerable attention. Filters designed by GA have the potential of obtaining near global optimum solution [5-6]. Although standard Gas (herein referred to as Real Coded GA (RGA)) have a good performance for finding the promising regions of the search space, they are inefficient in determining the local minimum in terms of convergence speed and solution quality. Particle Swarm Optimization (PSO) is an evolutionary algorithm developed by Kennedy and Eberhart in 1995 [8-9]. Several attempts have been made towards the optimization of the FIR Filter [14] and in other areas [10] also using PSO algorithm. The PSO is simple to implement and its convergence may be controlled via few parameters. This paper describes the FIR digital filter design using the PSO with constriction factor and inertia weight approach (PSO-CFIWA). PSO-CFIWA algorithm tries to find best coefficients that closely match the ideal frequency response. The rest of the paper is arranged as follows. In section 2, the filter design problem is formulated. Section 3 briefly discusses on the real coded GA (RGA). Section 4 shows the algorithms of GA, general PSO and PSO-CFIWA. Section 5 describes the simulation result. Finally section 6 concludes the paper.

II. Filter Design

A digital FIR filter is characterized by

$$H(z)=\sum_{n=0}^{N} h(n)z^{-n}\text{, n=0, 1, …, N,} \tag{1}$$

We assume that $h(n)\neq 0$ and $h(0)\neq 0$

Where, N is the order of the filter which has N+1 number of coefficients. h(n) is the filter impulse response. It is calculated by applying an impulse signal at the input. The value of h(n) will determine the type of the filter e.g. low pass, high pass, band pass etc. The value of h(n) is to be determined in the design process and N represents the order of the polynomial function. This paper presents the most widely used FIR that h(n) is odd symmetric and the order is even. The length of h(n) is N+1 and the number of coefficients is also N+1. The individual represents h(n) . In each iteration, these individuals are updated. Fitness of particles is calculated using the new coefficients. This fitness is used to improve the search

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in each iteration, and result obtained after a certain number of iterations or after the error is below a certain limit is considered to be the final result. Because its coefficients are matched, the dimension of the problem reduces by a factor of 2. The (N+1)/2 coefficients are then flipped and concatenated to find the required N+1 coefficient. The least square (LS) error is used to evaluate the individual. It takes the squared error between the frequency response of the ideal and the actual filter. An ideal filter has a magnitude of 1 on the pass band and a magnitude of 0 on the stop band. So the error for this fitness function is the squared difference between the magnitudes of this filter and the filter designed using the evolutionary algorithms. The individuals that have higher evaluation values represent the better filters, the filters with better frequency response. The frequency response of the FIR digital filter can be calculated as,

$$H\left(e^{jw_k}\right)=\sum_{n=0}^{N} h(n)e^{-jw_k n};$$

Where, $w_k = \frac{2\pi k}{N}$; $H\left(e^{jw_k}\right)$ is the Fourier transform complex vector. This is the FIR filter frequency response.
The expression of the LS function is given below:

$$Error = Min\sum_{k=1}^{K}\left|H_i\left(e^{j\omega_k}\right)-H_d\left(e^{j\omega_k}\right)\right|^2 \qquad (2)$$

Where, H_i represents the ideal magnitude response of the filter and is given as

$$\begin{aligned}H_i\left(e^{j\omega_k}\right)&=1 \, for \, 0\le\omega\le\omega_c\\&=0 \text{ otherwise}\end{aligned} \qquad (3)$$

$H_d\left(e^{j\omega_k}\right)$ represents the filter to be designed, K is the number of samples. Equation (2) represents the fitness function to be minimized using evolutionary algorithm.

III. Real Coded Genetic Algorithm (RGA)

GA is mainly a probabilistic search technique, based on the principles of natural selection and evolution. At each generation it maintains a population of individuals where each individual is a coded form of a possible solution of the problem at hand called chromosome. Chromosomes are constructed over some particular alphabet, e.g., the binary alphabet {0, 1}, so that chromosomes' values are uniquely mapped onto the decision variable domain. Each chromosome is evaluated by a function known as fitness function, which is usually the fitness function or the objective function of the corresponding optimization problem. Steps of RGA as implemented for optimization of spacing between the elements and current excitations are [10, 11]:

- Initialization of real chromosome strings of n_p population, each consisting of a set of excitations. Size of the set depends on the number of excitation elements in a particular array design.
- Decoding of strings and evaluation of *Error* of each string.
- Selection of elite strings in order of increasing *Error* values from the minimum value.
- Copying of the elite strings over the non-selected strings.
- Crossover and mutation to generate off-springs.
- Genetic cycle updating.
- The iteration stops when the maximum number of cycles is reached. The grand minimum *Error* and its corresponding chromosome string or the desired solution are finally obtained.

IV. Particle swarm Optimization (PSO)

PSO is a flexible, robust population-based stochastic search or optimization technique with implicit parallelism, which can easily handle with non-differential objective functions, unlike traditional optimization methods. PSO is less susceptible to getting trapped on local optima unlike GA, Simulated Annealing etc. Eberhart and Shi [9] developed PSO concept similar to the behavior of a swarm of birds. PSO is developed through simulation of bird flocking in multidimensional space. Bird flocking optimizes a certain objective function. Each agent knows its best value so far (pbest). This information corresponds to personal experiences of each agent. Moreover, each agent knows the best value so far in the group (gbest) among pbests. Namely, each agent tries to modify its position using the following information:

- The distance between the current position and pbest.
- The distance between the current position and gbest.

Mathematically, velocities of the particles are modified according to the following equation [11]:

$$\begin{aligned}V_i^{k+1} = w*V_i^k &+ C_1*rand_1*\left(pbest_i^k - S_i^k\right)\\ &+ C_2*rand_2*\left(gbest^k - S_i^k\right)\end{aligned} \qquad (4)$$

where V_i^k is the velocity of agent i at iteration k ; w is the weighting function; C_j is the weighting factor; $rand_i$ is the random number between 0 and 1; S_i^k is the current position of agent i at iteration k; $pbest_i^k$ is the pbest of agent i; $gbest^k$ is the gbest of the group. The searching point in the solution space can be modified by the following equation:

$$S_i^{k+1} = S_i^k + V_i^{k+1} \qquad (5)$$

The first term of (4) is the previous velocity of the agent. The second and third terms are used to change the velocity of the agent. Without the second and third terms, the agent will keep on "flying" in the same direction until it hits the boundary. w, corresponds to a kind of inertia and tries to explore new areas. For Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach (PSOCFIWA) [12, 13], the velocity of (4) is manipulated in accordance with (6).

$$\begin{aligned}V_i^{k+1} = CFa \times (w^{k+1}*V_i^k &+ C_1*rand_1*\left(pbest_i - S_i^k\right)\\ &+ C_2*rand_2*\left(gbest - S_i^k\right))\end{aligned} \qquad (6)$$

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Normally, C_1=C_2=1.5-2.05 and Constriction Factor (CFa) is given in (7).

$$CFa = \frac{2}{\left|2-\varphi-\sqrt{\varphi^2-4\varphi}\right|} \tag{7}$$

Where

$\varphi = C_1 + C_2$, and φ >4.

For==2.05, the computed value of = 0.73.

The best values of C_1, C_2, and are found to vary with the design sets. In inertia weight approach (IWA), inertia weight at $(k+1)^{th}$ cycle is as given in (8).

$$w^{k+1} = w_{max} - \frac{w_{max} - w_{min}}{k_{max}} \times (k+1) \tag{8}$$

Where, w_{max} =1.0; w_{min} =0.4; k_{max} = Maximum number of iteration cycles. The solution updating is the same as (5).

V. Results and Discussions

A. Analysis of Magnitude response of low-pass FIR filters

The MATLAB simulation has been performed extensively to realize the low pass FIR filter with the order of 20 and 30. Hence the length of the filter coefficients is 21 and 31, respectively. The sampling frequency was chosen as f_s = 1Hz. Also, for all the simulations the sampling number was taken as 128.

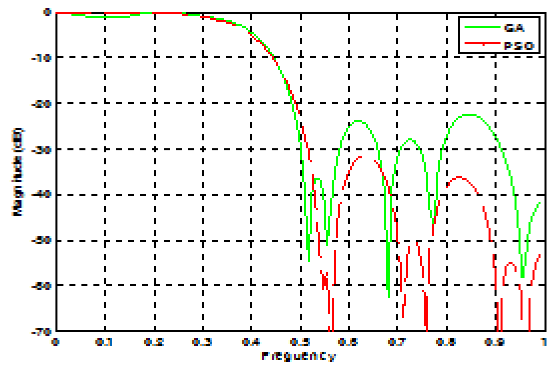
TABLE I
GA Parameters

Parameter	Value
Population Size	120
Crossover rate	1
Crossover	Two Point Crossover
Generation number	500
Mutation rate	0.01
Mutation	Gaussian Mutation
Selection	Roulette
Selection Probability	1/3

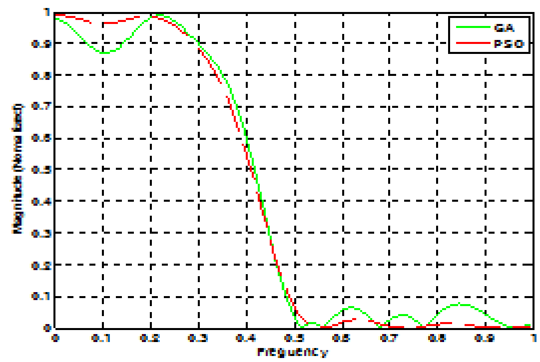
The control parameter values GA used in this work are given in Table I. Pass band normalized cut off frequency is 0.4. Algorithms are run for 25 times. The best possible sets of coefficients for the designed FIR filter for order 20 and 30 have been shown in Table II and Table III, respectively.

TABLE II.
Optimized Coefficients of FIR Filter of Order 20

H(N)	RGA	PSOCFIWA
H(1)=H(21)	0.0174	0.0072
H(2)=H(20)	0.0085	0.0012
H(3)=H(19)	-0.0164	-0.0094
H(4)=H(18)	0.0157	0.0037
H(5)=H(17)	0.0248	0.0220
H(6)=H(16)	0.0101	0.0054
H(7)=H(15)	-0.0533	-0.0488
H(8)=H(14)	-0.0632	-0.0509
H(9)=H(13)	0.0703	0.0784
H(10)=H(12)	0.2751	0.2878
H(11)	0.3982	0.3980



(a)



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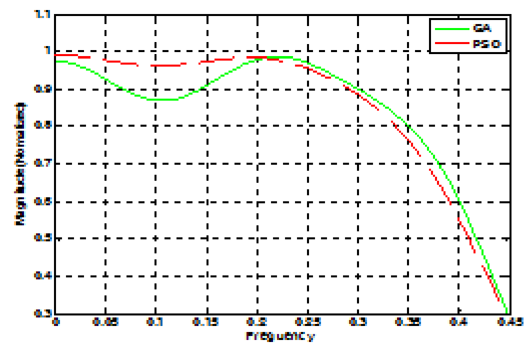

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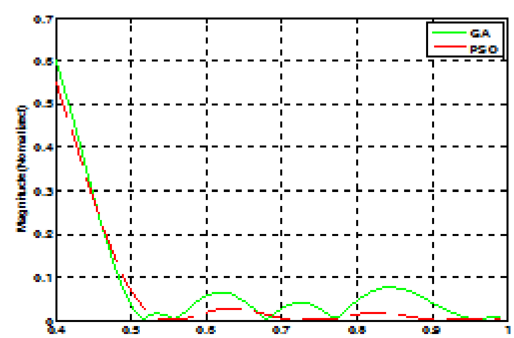

(d)

Fig. 1 Magnitude response of 20^{th} order low-pass FIR filters

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TABLE III. Optimized Coefficients of FIR Filter of Order 30

H(N)	RGA	PSOCFIWA
H(1)=H(31)	0.0058	0.0032
H(2)=H(30)	0.0052	0.0021
H(3)=H(29)	-0.0098	-0.0062
H(4)=H(28)	-0.0008	0.0005
H(5)=H(27)	0.0041	0.0049
H(6)=H(26)	0.0067	0.0037
H(7)=H(25)	-0.0171	-0.0152
H(8)=H(24)	-0.0168	-0.0144
H(9)=H(23)	0.0072	0.0107
H(10)=H(22)	0.0284	0.0307
H(11)=H(21)	0.0023	0.0014
H(12)=H(20)	-0.0638	-0.0637
H(13)=H(19)	-0.0555	-0.0561
H(14)=H(18)	0.0828	0.0856
H(15)=H(17)	0.2953	0.2971
H(16)	0.4003	0.4003

As seen from Figure 1, for the pass band region, the new PSO (PSO-CFIWA) produces a better response than that of GA. The filters designed by the PSO algorithm have sharper transition band responses than that produced by GA algorithm. For the stop band region, the filters designed by the PSO-CFIWA method produce better responses than the others. The best optimized coefficients obtained for the designed filter with the order of 20 and 30 have been calculated by the two methods and given in Table II and III, respectively.

B. Comparative effectiveness and convergence profiles of RGA and PSO-CFIWA

In order to compare the algorithms in terms of the convergence speed, Figure 3 shows the evolution of best solutions obtained when GA is employed. Figure 4 shows the evolution of best solutions obtained when the new PSO is employed. The convergence graph has been shown for the filter order of 30. A similar plot can be obtained for the FIR filter of order 20. From the figures drawn for this filter, it is seen that the PSOCFIWA algorithm is significantly faster than the GA algorithm for finding the optimum filter. The new PSO converges to a much lower fitness in lesser number of iterations. The minimum Error values are plotted against the number of iteration cycles to get the convergence profiles for the optimization techniques. Figs. 3-4 show the convergence profiles for RGA and PSOCFIWA for 30th order low-pass FIR filters respectively. RGA and PSOCFIWA converge to their respective minimum ripple magnitude in less than 500 iterations. Further RGA yields suboptimal higher values of Error but PSOCFIWA yields near optimal (least) Error values consistently in both cases. With a view to the above fact, it may finally be inferred that the performance of PSOCFIWA technique is better as compared to RGA. All optimization programs are written in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

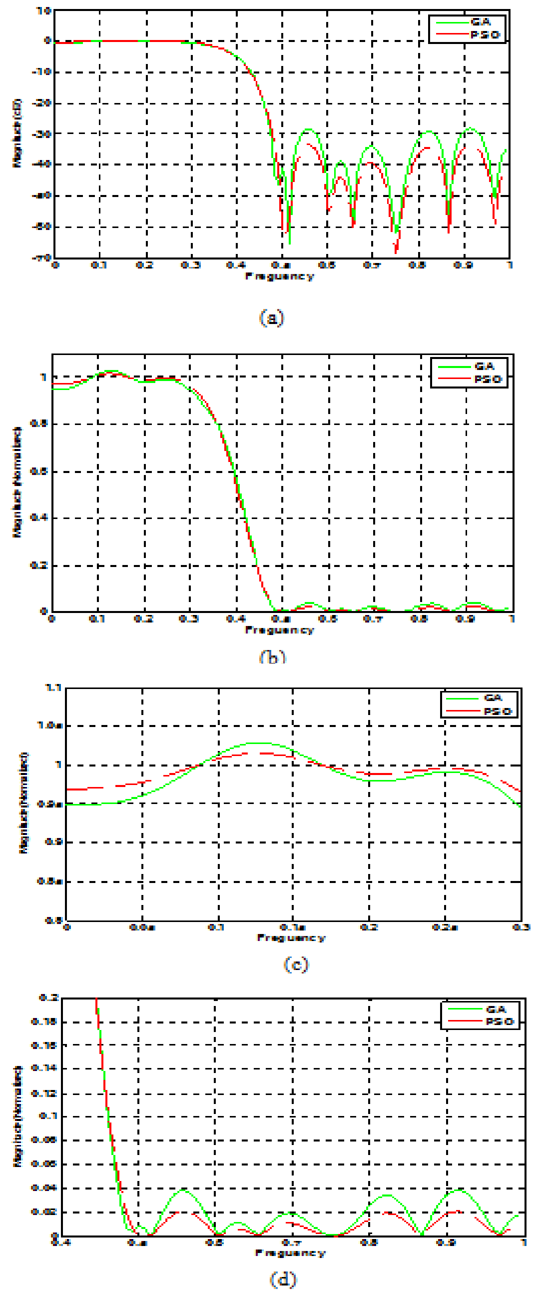

Fig. 2 Magnitude response of 30th order low-pass FIR filters

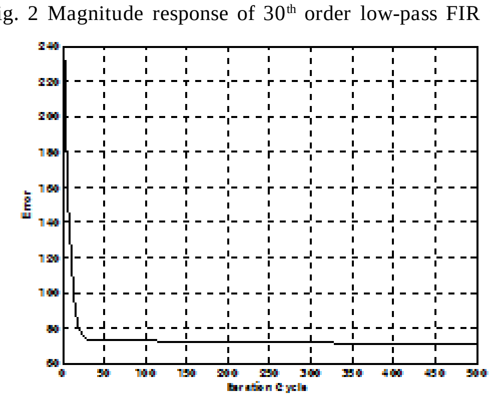

Fig. 3. Convergence profile for RGA in case of 30th order low-pass FIR filters.

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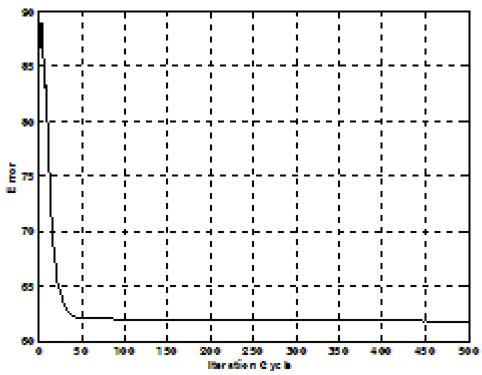

Fig. 4. Convergence profile for PSOCFIWA in case of 30th order low-pass FIR filters

VI. Conclusions

This paper presents an alternative approach for FIR filter deign using PSOCFIWA. Filters of orders 20 and 30 have been realized using GA as well as PSOCFIWA. Extensive simulation results justify that the proposed algorithm outperforms GA in the accuracy of the magnitude response of the filter as well as in the convergence speed.

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